

Interpreting the macroscopic pointer by analysing the Einstein-Podolsky-Rosen steering of an entangled macroscopic superposition state

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We examine Einstein-Podolsky-Rosen's (EPR) steering nonlocality for two realisable Schrodinger cat-type states where a meso/ macroscopic system (called the "cat"-system) is entangled with a spin-1/2 system. For large cat-systems, we show that a local hidden state model is near-satisfied, meaning that the cat-system can be consistent with being in a mixture of "dead" and "alive" states despite that it is entangled with the spin system. We justify that a rigorous signature of the Schrodinger cat-type paradox is the EPR-steering of the cat-system and provide two experimental signatures. This leads to a hybrid quantum/ classical interpretation of the macroscopic pointer of a measurement device and suggests many Schrodinger cat-type paradoxes can be explained by microscopic nonlocality.

The original arguments of Einstein-Podolsky-Rosen (EPR) and Bell dealt with small symmetrical systems: two particles or two spins [1, 2]. The arguments are based on EPR's notion of local realism (LR) – put simply, that there can be no "spooky action-at-a-distance" [3] on one system as a result of measurements made on the other. In revealing inconsistencies between the predictions of quantum mechanics and the premise of local realism (LR), these arguments have had profound implications for physics [4]. Schrodinger recognised that the consequences of such paradoxes would be significant for larger systems [5]. He analysed a quantum gedanken experiment whereby a macroscopic system C (likened to a cat and that we refer to as the "cat-system") becomes entangled with a microscopic spin 1/2 system S , the final state being the superposition

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|A\rangle_C |\uparrow\rangle_S + |D\rangle_C |\downarrow\rangle_S) \quad (1)$$

Here, $|A\rangle_C$ and $|D\rangle_C$ represent macroscopically distinguishable states in which the "cat" is "dead" (if given by $|D\rangle_C$) or "alive" (if given by $|A\rangle_C$). The $|\uparrow\rangle$, $|\downarrow\rangle$ are the eigenstates of the Pauli spin σ_Z . The spin and "cat" systems can in principle become spatially separated.

While Schrodinger pointed out the natural interpretation of this state – that the "cat" cannot be viewed as either "dead" or "alive" until measured – he did not construct an EPR-type experiment that would *demonstrate* such failure of reality for a practical realisation of (1). While such signatures have since been developed (for example [6–10]), experimental work has mainly focused on providing evidence (such as a fidelity or entanglement measure) for the state (1) within quantum theory [11–16] and the signatures do not directly examine the reality of the cat-system C itself, as distinct from that of the spin-system S . Yet, understanding the precise nature of the failure of classical realism for the state (1) is of topical interest: Many proposals have been put forward so that the paradoxical situation in which a "cat" is apparently 'both alive and dead' can be better understood [17–19].

The objective of this Letter is to probe the asymmetrical nature of the entanglement of the superposition state

(1) by way of an EPR paradox (called an EPR steering paradox in this more general situation [21]). The important feature of our analysis is that EPR's local realism is defined *asymmetrically*, so one may consider either "spooky" action on the cat-system C by measurements on the spin-system S , or vice versa [1]. This opens up the possibility that nonlocality (which is the negation of LR) can manifest asymmetrically between the two systems [22, 23]. That EPR-steering can be detectable one-way but not the other has recently been confirmed experimentally for qubit systems [24].

In this Letter, we utilise this feature to gain an understanding of the discrepancy between the quantum and classical descriptions (which we call the degree of "*quantumness*") for *each* of the sub-systems (the "cat" C and the spin S) of the state (1). This information is not given by the observation of entanglement alone. It would be expected that this discrepancy can be different for the two sub-systems. We find this is indeed the case, but are further able to show that the quantumness of the cat-system can approach zero, to the point where surprisingly the cat-system can be described as "dead" or "alive", despite that the two systems remain entangled. In this limit, the cat-system acts as a *classical measuring device* for the microscopic system, which maintains its quantumness.

This motivates the question of how to determine when the cat-system itself is paradoxical, along the lines suggested by Schrodinger. Such a bound is not set at the realisation of entanglement, but (we show) is set by the realisation of an EPR steering *of the cat-system*. This type of EPR steering manifests as a falsification of certain hidden states *for the cat-system*, that are implied by the premise of LR. These hidden states (or "elements of reality", as EPR called them) *predetermine* results for measurements on the "cat". In this paper, we calculate details of such elements of reality for two realisations of (1) one involving coherent states and the other Greenberger-Horne-Zeilinger (GHZ) spin states. By revealing contradictions, we thus arrive at measurable signatures for the EPR steering of the cat-system for two experimentally realisable mesoscopic superposition states. We confirm

that as the cat-system becomes larger, the hidden states become indistinguishable from classical states (in which the cat-system is “alive” or “dead”). In this limit, the EPR steering of the cat-system is eventually lost, but we verify that EPR steering of the spin by the cat-system can be maintained, so that measurements on the cat-system can certify the quantumness of the spin.

The regime where the cat-system is large is particularly interesting, since in this limit the cat-system models a pointer of a measuring device for σ_Z of the spin. Measurement paradoxes have been raised in the literature, because the interpretation of the “cat” being both “alive” and “dead” is then that the “pointer” is at two (macroscopically separated) positions of a dial at once. While decoherence mechanisms preclude such a result, it is interesting nonetheless to understand the entangled state *without decoherence* and how the collapse of the pointer into a state of “one position *or* the other” occurs. Our results indicate that the “*quantum pointer*” regime is very nearly a regime of one-way steering (or entanglement) where the cat-system is fully classical. We discuss how this suggests a hybrid quantum-classical picture of the *pointer*, that immediately after interaction with the microscopic system, the pointer is located at (near) one of the two macroscopically distinct positions, but with an indeterminacy related to nonlocal effects bounded in size by the uncertainty relation.

Coherent cat-states: Consider the following well-known prototype for the superposition state (1) [11]:

$$|\psi_{coh}\rangle = \frac{1}{\sqrt{2}} \left(e^{-i\pi/4} |\alpha\rangle |\uparrow\rangle_Z + e^{i\pi/4} |-\alpha\rangle |\downarrow\rangle_Z \right) \quad (2)$$

Here $|\alpha\rangle$ is a coherent state for a quantum harmonic oscillator system that we refer to as the “cat”-system C . The $|\uparrow\rangle_Z, |\downarrow\rangle_Z$ are eigenstates of σ_Z for the spin system S . We take α to be real and (ideally) large. Observers Alice and Bob can make measurements on the spin and cat-systems respectively. We consider that the two systems have become spatially separated after the interaction that created the entanglement. If Alice measures σ_Z and the result is 1, then the state of system C is $|\alpha\rangle$. Similarly, if the result is -1 , the state is $|-\alpha\rangle$. Thus, Alice can *predict* the statistics for Bob’s measurements, conditional on her outcome. Suppose Bob makes a measurement of either the position or momentum quadrature defined (in a rotating frame) by $X = \frac{1}{\sqrt{2}}(a^\dagger + a)$ and $P = \frac{i}{\sqrt{2}}(a - a^\dagger)$. Here a^\dagger, a are the creation, destruction operators for system C . If Alice’s outcome is ± 1 , the conditional probability distribution $P(x)$ for the outcome x of Bob’s measurement X in each case is the Gaussian hill

$$P_{\pm}(x) = \frac{1}{\sqrt{\pi}} \exp\{-(x \mp \sqrt{2}\alpha)^2\} \quad (3)$$

centred at $\pm\sqrt{2}\alpha$ respectively and with variance $(\Delta x)^2 = 1/2$ as for a coherent state. The \pm hills are distinguished as “alive” and “dead” for large α .

EPR postulated that the measurement by Alice makes no difference to the system of the other observer. Bell’s expression of LR is that the joint probability $P_{CS}(x, y)$ for outcomes x and y of measurements made at C and S respectively can be described by a Local Hidden Variable (LHV) model such that [2, 4, 17]

$$P_{CS}(x, y) = \int_{\lambda} d\lambda \rho(\lambda) P_C(x|\theta, \lambda) P_S(y|\phi, \lambda) \quad (4)$$

Here λ symbolises a set $\{\lambda\}$ of hidden variables that have a distribution $\rho(\lambda)$; ϕ and θ are the measurement choices for S and C respectively. The locality assumption is that $P_C(x|\theta, \lambda)$ is independent of Alice’s measurement choice ϕ (for spin) and the outcome y at location S ; similarly $P_S(y|\phi, \lambda)$ is independent of Bob’s choice θ and result x at C . We note there is an *asymmetry* in the locality assumption for the EPR experiment, because the measurements x and θ by Bob are spacelike separated from those of Alice but are in the future [26]. We call this premise LR $S \rightarrow C$. Of special interest to us is where an extra constraint is put on the $P_C(x|\theta, \lambda)$ that they be consistent with the statistics arising from a local quantum state i.e. that there exists a quantum density operator $\rho_{C,\lambda}$ that predict the probabilities $P_C(x|\theta, \lambda)$. Such probabilities are denoted with a subscript q , and the model becomes the Local Hidden State (LHS) model of Ref. [21]

$$P_{CS}(x, y) = \int_{\lambda} d\lambda \rho(\lambda) P_C(x|\theta, \lambda)_q P_S(y|\phi, \lambda) \quad (5)$$

the falsification of which is *certification of EPR-steering of the cat-system C*.

EPR noticed that the assumption of LR and a strong statistical correlation between two systems S and C place restrictions on the hidden variables λ and the predictions $P_C(x|\theta, \lambda)$ given in (4). We find that the local cat-system must be consistent with being in a mixture of hidden states that predetermine the cat-system to be either “dead” or “alive”. This is expressed as the following result, proved in the Supplemental Materials [26].

Result (1a): Given LR (as the LHV model (4)), the local cat-system C is consistent with being *either* in a (hidden-variable) state with the distribution for X given by $P_+(x)$ (“alive”), *or* in a state with statistics given by $P_-(x)$ (“dead”). This implies that the hidden variable set $\{\lambda\}$ includes a variable λ_Z , which defines the two predetermined states by $\lambda_Z = +1$ or -1 respectively. EPR used the term “elements of reality” to describe the predetermination.

To show EPR-steering, we consider that Alice measures σ_X [27]. Alice is able to predict the probability distribution for Bob’s measurement P on the system C , conditional on her outcome ± 1 . The conditional distribution $P(p)$ for P is

$$P_{\pm}(p) = \frac{1}{\sqrt{\pi}} \exp(-p^2) (1 \pm \sin(2\sqrt{2}p\alpha)) \quad (6)$$

The distribution exhibits interference fringes and has a variance $(\Delta p)^2 = \frac{1}{2} - 2\alpha^2 e^{-4\alpha^4}$ reduced *below* that of the coherent state, for which $(\Delta p)^2 = \frac{1}{2}$. Result (1a) leads to the conclusion the cat-system C is in one or other of two states, that correspond to the distributions $P_+(p)$ and $P_-(p)$ respectively. We denote these hidden states by the variable λ_X , which assumes the value $+1$ or -1 in each case. For consistency with the LHV model (4), we show ((**Result 1b**) in the Supplemental Materials) that the local cat-system C would *simultaneously* be described by both variables: λ_Z and λ_X . We represent such an element of reality state by the ordered pair (λ_Z, λ_X) .

Now we note the inconsistency that gives an EPR steering paradox. There are *four* element of reality states of the cat, as depicted in Figure 1: each (λ_Z, λ_X) has predictions for X and P given by $P_{\lambda_Z}(x)$ and $P_{\lambda_X}(p)$ respectively. We see that for each of these states

$$\Delta X \Delta P = \frac{1}{2} (1 - 4\alpha^2 e^{-4\alpha^4})^{1/2} < \frac{1}{2} \quad (7)$$

which contradicts the Heisenberg uncertainty relation $\Delta X \Delta P \geq \frac{1}{2}$. Thus, the element of reality states cannot be *quantum* states: The inequality (7) is the EPR steering inequality $\Delta_{inf} X \Delta_{inf} P < 1/2$ where $(\Delta_{inf} X)^2 = \sum_{\sigma_Z} P(\sigma_Z) (\Delta(X|\sigma_Z))^2$ and $(\Delta_{inf} P)^2 = \sum_{\sigma_X} P(\sigma_X) (\Delta(P|\sigma_X))^2$ are the average inference variances for X and P . Here, $P(\sigma_Z)$ is the probability of outcome σ_Z for σ_Z and $(\Delta(X|\sigma_Z))^2$ is the variance of the conditional distribution $P(X|\sigma_Z)$. This inequality signifies the failure of all LHS models (5) and hence an *EPR steering of the cat-system* [21, 25, 27].

In other words, the inequality (7) negates that the local cat-system C is in any mixture of any “dead” or “alive” (*local*) quantum states as consistent with the LHV model (4). This is proved for all α . However, as $\alpha \rightarrow \infty$, the falsification of the LHS model (5) (evident by the fringe pattern) becomes unverifiable. This is shown in Figure 2, where for $\alpha \sim 100$, the LHS model cannot be falsified visually given the finite resolution of the graphics.

The main point of this paper is that where the LHS model (5) is not falsifiable, there can be *no demonstration of the loss of classical reality of the cat-system itself*. This is because the expression (5) describes the cat-system being in a classical mixture of the local hidden quantum states $\rho_{C,\lambda}$ consistent the hidden variable λ_Z and therefore being in quantum states either “dead” or “alive”. It is known that the LHS model (5) can hold, despite that the two systems are entangled [21]. Entanglement is certified by negating the quantum separable model where the predictions $P_S(y|\phi, \lambda)$ are also constrained to be consistent with a quantum density operator. In short, entanglement can be confirmed based on a strong nonclassicality of the spin system S , regardless of the quantumness of the cat-system, and is a less rigorous measure of the cat-paradox.

From (2) we see that as $\alpha \rightarrow \infty$, measurement of X is also a measurement of spin σ_Z . The EPR steering of the

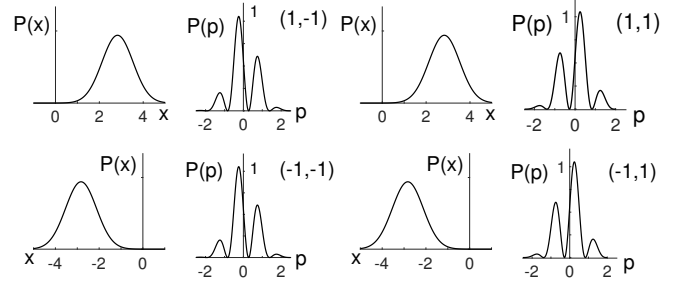


Figure 1. Predictions $P(x)$ and $P(p)$ for “element of reality” states (λ_Z, λ_X) of the coherent cat-state (2) with $\alpha = 2$. Top: The “alive” cat-system with $(1, -1)$ or $(1, 1)$. Lower: The “dead” cat-system with $(-1, -1)$ or $(-1, 1)$.

spin-system S can be realised by number measurements on system C that distinguish between adjacent odd and even values. Thus, operational regimes of genuine one-way steering in which a fully classical (“dead” or “alive”) cat-system can detect of the “quantumness” of the spin (but not vice versa) are viable. The full details are given in the Supplemental Materials [26].

GHZ states: Similar results are achieved for a second realisation of the state (1). The GHZ state $|\psi_{GHZ}\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle^{\otimes N} - |\downarrow\rangle^{\otimes N})$ is formed from N spin-1/2 particles [14, 15, 28]: Here $|\uparrow\rangle^{\otimes N} = \prod_{k=1}^N |\uparrow\rangle^{(k)}$ and $|\uparrow\rangle^{(k)}$ is the spin eigenstate for $\sigma_Z^{(k)}$, the σ_Z observable for the k -th particle. If we separate the N -th spin from the remaining $N-1$ spins, the GHZ state is a microscopic spin S entangled with a larger system C similar to (1):

$$|\psi_{GHZ}\rangle = \frac{1}{\sqrt{2}} \left(|\uparrow\rangle_C^{\otimes N-1} |\uparrow\rangle^{(N)} - |\downarrow\rangle_C^{\otimes N-1} |\downarrow\rangle^{(N)} \right) \quad (8)$$

Alice makes a measurement on the single spin, while Bob measures the “cat”-system C of $N-1$ particles. We define the collective spin for the system C as $\sigma_Z^B = \sum_{k=1}^{N-1} \sigma_Z^{(k)}$. The measurement of $\sigma_Z^{(N)}$ by Alice will reduce the cat-system into the “alive” state $|\uparrow\rangle_C^{\otimes N-1}$ if her result is $+1$, or to the “dead” state $|\downarrow\rangle_C^{\otimes N-1}$ if her result is -1 . Assuming LR and Result 1, the cat-system is deduced to be “alive” or “dead” i.e. always in one *or* the other of two element of reality states that correspond to the outcomes $\pm(N-1)/2$ for σ_Z^B respectively. We denote these respective states by a hidden variable λ_Z with values ± 1 . To realise EPR steering, we consider that Alice measures $\sigma_X^{(N)}$. We choose $N = 3, 7, \dots$. A measurement of $\sigma_X^{(N)}$ gives the result 1 or -1 which predicts precise outcomes for Bob’s $Pr_Y^B = \prod_{k=1}^{N-1} \sigma_Y^{(k)}$. Assuming LR, Result 1 implies system C to be in one of the element of reality states specified by a hidden variable λ_X , where the value $\lambda_X = \pm 1$ corresponds to outcomes for Pr_Y^B being ± 1 . Suppose Alice measures $\sigma_Y^{(N)}$. Assuming LR, the system C is also in an element of reality state denoted by a third hidden variable λ_Y where the value $\lambda_Y = \pm 1$ corresponds to

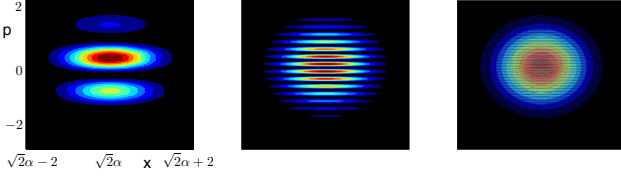


Figure 2. *Quantum-classical transition*: Predictions of the element of reality state (1,1) for the cat-system of (2) with (from left) $\alpha = 2, 10, 100$. Plotted are contours for $P_1(x)P_1^{(X)}(p)$ versus x (horizontal) and p (vertical). As $\alpha \rightarrow \infty$ the element of reality cannot be resolved as distinguishable from the classical description $|\alpha\rangle$.

outcomes for all products $Pr_Y^B(J) = \sigma_X^{(J)} \prod_{k=1, k \neq J}^{N-1} \sigma_Y^{(k)}$, $J = 1, \dots, N-1$ being ± 1 . Therefore, Result 1b implies the cat-system C to be in one of the simultaneous element of reality states $(\lambda_Z, \lambda_X, \lambda_Y)$ in which the outcomes for $\sigma_Z^B, Pr_Y^B, Pr_Y^B(J)$ are each predetermined with no uncertainty. Yet, observables $\sigma_Z^B, Pr_Y^B, \sum_{J=1}^{N-1} Pr_Y^B(J)$ satisfy the Heisenberg uncertainty relation

$$\Delta(\sigma_Z^B) \Delta(Pr_Y^B) \geq |\langle \sum_{J=1}^{N-1} Pr_Y^B(J) \rangle|/2 \quad (9)$$

The hidden states $(\lambda_Z, \lambda_X, \lambda_Y)$ (which specify predetermined *nonzero* values for each of these observables) contradict (9). As with inequality (7) this contradiction signifies an EPR-steering of the cat-system and negates any mixture of (local) quantum “dead” and “alive” hidden states consistent with LR [26].

So far, there is no falsification that the cat-system can be described as a mixture of “dead” or “alive” *hidden variable* states. Such a falsification is achieved if the full LHV model (4) can be violated for the cat-state. This is predicted possible using Svetlichny inequalities [6].

Discussion: The EPR steering signatures derived in this paper falsify that the local cat-system can be in a “dead” or “alive” quantum state *consistent* with the full separability of the LHS model i.e. assuming no nonlocal effects between the “cat”- and spin systems. Nonlocality has been verified in recent experiments but such nonlocal effects are small, corresponding to predictions of \sim one spin unit. A relevant question is whether such small nonlocal effects could explain the “cat”-paradox in the context of the state (1). Our results suggest this cannot be ruled out: If we allow that there could be microscopic nonlocality, then the steering signatures of the cat-states (2) and (3) vanish. To discuss this, we *quantify* the LR premise. We define that for δ -scopic LR $S \rightarrow C$, it is assumed that Alice’s measurement of the spin does not affect the (value of measurement on the) cat-system by an amount more than δ . Hidden variables for the cat-system can then be defined with different amounts of indeterminacy δ in the prediction for measurements (due to different amounts of allowed nonlocal change δ).

Result (2a): The EPR steering signatures are a negation of a fully separable LHS model (5). We can determine a value of δ such that if we allow nonlocality by an amount greater than δ , then the cat-system becomes indistinguishable from the classical mixture. We find δ (a measure of discrepancy between the quantum and classical descriptions) is classifiable as microscopic.

To explain, the EPR steering manifests in the cat-state (2) at large α through very fine fringes in distributions for P . One needs only relax the full locality condition to allow δ -scopic LR, where δ is a very small change in P , to nullify the steering. For the GHZ state, the EPR steering is lost when δ is a single spin unit. This ultra-sensitivity is consistent with proven fundamental requirements for signifying macroscopic quantum superpositions [19, 29].

Moreover, we see from Figure 2 that as $\alpha \rightarrow \infty$ the signature requires an increasingly stricter form of LR i.e. the value δ becomes smaller. This explains (similar to Refs. [19]) the fragility to decoherence as the size α of the cat-state increases – the “Schrodinger cat”-like behaviour is more difficult to observe because the elements of reality (which give a predetermination of the results of measurement) are closer to classically consistent values.

Hence it is the hidden variables/ states for the cat-system that specify outcomes of measurement to a *microscopic precision* that are negated by the EPR-steering signatures. The hidden variable λ_Z (that predetermines the outcome for the measurement X or σ_Z^B distinguishing the cat-system to be either “alive” or “dead”) can be defined with a macroscopic indeterminacy Δ , in which case we refer to it as a *macroscopic hidden variable* $\tilde{\lambda}_Z$. Because the “dead” and “alive” states are macroscopically separated, the hidden variable $\tilde{\lambda}_Z$ can still predetermine the cat-system to be “dead” or “alive” even for large Δ , though without full specification of the microscopic details of the prediction. The variable $\tilde{\lambda}_Z$ requires the assumption of Δ -scopic LR, but this is implied by δ -scopic LR where $\delta < \Delta$. The Result (2a) indicates that for most typical Schrodinger cat-type scenarios, the macroscopic hidden variable λ_Z *cannot be negated*. We can quantify with the following [26].

Result (2b): Suppose the uncertainty relation for X and P is $(\Delta X)(\Delta P) \geq c$ where c is a constant. Suppose we assume Δ -scopic LR and δ -scopic LR to deduce the hidden variables for measurement X and P respectively. If $\Delta\delta \geq c$, we cannot signify the cat-paradox by negation of the LHV model based on X, P measurements. \square This means that if we allow nonlocal effects of order $\gtrsim c$, the signature of the cat-paradox is lost. As $\alpha \rightarrow \infty$, the macroscopic element of reality $\tilde{\lambda}_Z$ predetermining the cat-system to be “dead” or “alive” can be defined with $\Delta \rightarrow \infty$. But then δ needs to be increasingly smaller.

Conclusion: For typical scenarios modelling the macroscopic entangled state (1), we cannot falsify the macroscopic element of reality $\tilde{\lambda}_Z$ for the cat-system. The interpretation of the cat-system being “both dead

and alive” becomes debatable in this context. This is clear because $\hat{\lambda}_Z$ is precisely the variable that *predetermines* the outcome of the *measurement* distinguishing whether the cat-system is “dead” or “alive”. We can however falsify (by EPR steering inequalities) that the cat-system is predetermined to be in a “dead” or “alive” *local hidden state*, where that hidden state has microscopic predictions independent of measurements made on the spin system (as in the LHS model (5)). If the cat-system is a measuring-device pointer, then an Ockham’s Razor interpretation is illustrated by Figures 2b and c. The pointer is positioned at one place on the dial *or* the other (as determined by the macroscopic element of reality $\hat{\lambda}_Z$ that cannot be negated) but with its position / momentum microscopically indeterminate due to microscopic nonlocality (illustrated by the fringes that reveal the falsification of the LHS model). On the other hand, we note from Result 2b that the quantification of the relevant nonlocal effect is given by c which more generally need not be considered microscopic [30].

The cat-states we describe can be realised for a mechanical oscillator coupled to a two-level atom or optical system and for a microwave field mode coupled to Rydberg atoms [11, 14]. Photonic states have been reported with Svetlichny-Bell violations [12, 15]. The steering signatures are thus likely measurable by experiment.

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